

Aichelburg-Sexl boost of domain walls and cosmic strings

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We consider the application of the Aichelburg-Sexl boost to plane and line distributions of matter. Our analysis shows that for a domain wall the space-time after the boost is flat except on a null hypersurface which is the history of a null shell. For a cosmic string we study the influence of the boost on the conical singularity and give the new value of the conical deficit.

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I. INTRODUCTION

The properties of the space-time which results from boosting a spherical mass to the velocity of light were first studied by Aichelburg and Sexl [1]. This was later extended to the Kerr geometry [2,3] and to a static axially symmetric source characterized by its multipole moments [4]. In all those cases the space-time after the boost is flat except on a singular null hypersurface. The Riemann tensor vanishes everywhere except on this surface, its nonvanishing components have a δ -type singularity whose value depends on the mass of the boosted particle [2] or on the multipole moments of the boosted source [4]. Furthermore the singular null hypersurface which is produced by the boost represents the history of a plane impulsive gravitational wave and admits a singular line along one of its null generators. In all these examples the Aichelburg-Sexl boost produces a change of the algebraic type of the Weyl tensor.

The properties of singular null hypersurfaces, i.e. space-times with a Riemann tensor having a δ -type singularity with support on a null hypersurface, have been studied already [5,6]. It has been shown that in general a null shell and an impulsive gravitational wave co-exist and that the singular null hypersurface represents the history of both the shell and the wave. One may then ask the following question: Can one find sources for which the Aichelburg-Sexl boost produces a null shell in addition to, or in place of, an impulsive gravitational wave?

In this work we consider two examples, one corresponding to a plane distribution of matter and the other to a line source. Planar sources have received considerable attention in the recent past in the context of brane cosmology. We have chosen here to study the effect of the Aichelburg-Sexl boost on planar domain walls both in vacuum and in (anti) de Sitter space, and with arbitrary dimension. Generalization to walls

with different equations of state would be straightforward from our analysis. We also consider the case of a straight cosmic string. An interesting aspect of this case is that the initial space-time possesses a conical singularity and one may wonder how this geometrical property is affected by the boost.

The Aichelburg-Sexl boost has been utilized in the study of the dynamics of compact objects at extreme energies such as for instance the scattering of ultrarelativistic black holes [7]. More recently it has received much attention in a classical model for high-energy scattering in quantum gravity which offers the exciting prospect that black holes could be produced in future accelerators. For this and other points of view see [8].

II. BOOSTING DOMAIN WALLS

Domain walls are two-dimensional topological defects in ordinary space and are characterized by an equation of state for which the surface energy density is equal to minus the surface pressure. The space-time model in general relativity of the gravitational field of such a source has been given by Vilenkin [9] for planar walls. It is conformally flat, locally flat (except on the hypersurface $z=0$, the history of the wall) and all slices $z=\text{const}$ are isometric to de Sitter space.

We first generalize the Vilenkin solution to $(n+1)$ -dimensional space-time M . The domain wall is thus an $(n-1)$ -dimensional surface and its history is a timelike n -dimensional hypersurface in M . We will consider only planar domain walls and start with a line element for M having the form

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = e^{2V(z)} \gamma_{ab}(x^c) dx^a dx^b + dz^2. \quad (2.1)$$

Here Greek indices take values $0, 1, 2, \dots, n$ while Latin indices take values $0, 1, 2, \dots, n-1$ and $x^n = z$. Also V is a function of z only and $\gamma_{ab} = \gamma_{ba}$ are functions of x^c only. The components of the Riemann curvature tensor for the metric given via the line element (2.1) are

$$R_{abcd} = e^{2V} R_{abcd}(\gamma) + e^{4V} V'^2 (\gamma_{ad} \gamma_{bc} - \gamma_{ac} \gamma_{bd}), \quad (2.2)$$

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$$R_{abcn}=0, \quad (2.3)$$

$$R_{ancn} = -e^{2V}(V'' + V'^2)\gamma_{ac}, \quad (2.4)$$

where $R_{abcd}(\gamma)$ are the components of the Riemann curvature tensor calculated with the metric tensor γ_{ab} . The Ricci tensor components for the metric tensor $g_{\alpha\beta}$ given above are

$$R_{ab} = R_{ab}(\gamma) - e^{2V}(V'' + nV'^2)\gamma_{ab}, \quad (2.5)$$

$$R_{an} = 0, \quad (2.6)$$

$$R_{nn} = -n(V'' + V'^2). \quad (2.7)$$

If we now require that the space-time M be conformally flat then the vanishing of the Weyl tensor for the metric $g_{\alpha\beta}$ results in

$$R_{abcd}(\gamma) = \frac{R(\gamma)}{n(n-1)}(\gamma_{ac}\gamma_{bd} - \gamma_{ad}\gamma_{bc}). \quad (2.8)$$

Hence the slices $z=\text{const}$ have constant curvature $R(\gamma)/n(n-1)$ provided $n>2$. The Einstein tensor calculated with γ_{ab} is given by

$$G_{ab}(\gamma) = \left(\frac{2-n}{2n}\right)R(\gamma)\gamma_{ab}. \quad (2.9)$$

The Einstein tensor calculated with the metric $g_{\alpha\beta}$ of M has nonidentically vanishing components

$$G_b^a = e^{-2V}G_b^a(\gamma) + (n-1)\left[V'' + \frac{n}{2}V'^2\right]\delta_b^a, \quad (2.10)$$

$$G_n^n = -\frac{1}{2}e^{-2V}R(\gamma) + \frac{1}{2}n(n-1)V'^2. \quad (2.11)$$

Substituting Eq. (2.9) into Eq. (2.10) and eliminating $R(\gamma)$ from the resulting equation using Eq. (2.11) we arrive at

$$G_b^a = \left[\frac{1}{2}(n-2)G_n^n + (n-1)(V'' + V'^2)\right]\delta_b^a. \quad (2.12)$$

With these preliminaries completed we shall now consider the two cases of the history of a domain wall in a vacuum space-time and in (A)dS space-time.

For a domain wall in a vacuum ($G_\beta^\alpha=0$) the above equations give the following equation for $V(z)$:

$$V'' + V'^2 = 0. \quad (2.13)$$

The general solution is

$$V(z) = \ln(cz + 1), \quad (2.14)$$

where c is a constant of integration. The second constant of integration has been put equal to unity without loss of generality. However a planar domain wall with $z=0$ as its history in the space-time M must be reflection symmetric in $z=0$. To achieve this we adjust Eq. (2.14) to read

$$V(z) = \ln(c|z| + 1), \quad (2.15)$$

which now, of course, no longer satisfies Eq. (2.13) on $z=0$ but satisfies

$$V'' + V'^2 = 2c\delta(z), \quad (2.16)$$

where $\delta(z)$ is the Dirac delta function which is singular on $z=0$. In addition the Einstein tensor of M no longer vanishes throughout M . The nonidentically vanishing components are

$$G_b^a = 2c(n-1)\delta(z)\delta_b^a. \quad (2.17)$$

It thus follows that $z=0$ is the history in M of a domain wall with surface energy density σ and surface pressure P given by

$$\sigma = -P = -\frac{c}{4\pi}(n-1). \quad (2.18)$$

In order to have $\sigma>0$ we must require $c<0$. It follows now from Eq. (2.10) that

$$G_b^a(\gamma) = -\lambda\delta_b^a, \quad (2.19)$$

with

$$\lambda = \frac{1}{2}(n-1)(n-2)c^2 > 0. \quad (2.20)$$

Hence each hypersurface $z=\text{const}$ in M is isometric to de Sitter space.

Let us now put $x^a = (x^0, x^i)$ with $x^0 = t$ and $i = 1, 2, 3, \dots, n-1$. The line element of M takes the form

$$ds^2 = (c|z| + 1)^2[-dt^2 + e^{-2ct}dx^i dx^i] + dz^2. \quad (2.21)$$

If we transform z to a new transverse coordinate Z on either side of $z=0$ such that $c|z| + 1 = e^{c|Z|}$ then this line element becomes

$$ds^2 = e^{2c|Z|}(-dt^2 + dZ^2) + e^{-2c(t-|Z|)}dx^i dx^i. \quad (2.22)$$

This generalizes to $(n+1)$ -dimensional space-time the solution of Vilenkin [9]. The non-identically vanishing components of the Einstein tensor of M now read

$$G_b^a = 2c(n-1)\delta(Z)\delta_b^a. \quad (2.23)$$

Now make the Lorentz boost in the Z direction,

$$\begin{aligned} t &= \bar{t} \cosh \chi - \bar{Z} \sinh \chi, \\ Z &= \bar{Z} \cosh \chi - \bar{t} \sinh \chi, \quad \bar{x}^i = x^i. \end{aligned} \quad (2.24)$$

The Lorentz factor of the boost is $\cosh \chi$. It is convenient to rewrite this transformation in terms of the two null coordinates $\bar{u} = \bar{t} - \bar{Z}$ and $\bar{v} = \bar{t} + \bar{Z}$ so that

$$t = \frac{\bar{u}}{2}e^\chi + \frac{\bar{v}}{2}e^{-\chi}, \quad Z = -\frac{\bar{u}}{2}e^\chi + \frac{\bar{v}}{2}e^{-\chi}. \quad (2.25)$$

Following [1] this boost is first applied to the line element (2.22) and then we take the limit in which the velocity of the boost tends to the speed of light ($\chi \rightarrow +\infty$). This results in Eq. (2.22) becoming

$$ds^2 = -e^{2\bar{c}|\bar{u}|} d\bar{u} d\bar{v} + e^{-2\bar{c}(\bar{u}-|\bar{u}|)} d\bar{x}^i d\bar{x}^i, \quad (2.26)$$

where the parameter \bar{c} is related to the parameter c in Eq. (2.22) via

$$\bar{c} = c \cosh \chi. \quad (2.27)$$

The transformed Einstein tensor $\bar{G}_{\alpha\beta}$ vanishes except for

$$\bar{G}_{\bar{u}\bar{u}} = 2\bar{c}(n-1)\delta(\bar{u}). \quad (2.28)$$

When $\bar{u} \neq 0$ the line element (2.26) can be transformed into Minkowskian form

$$ds^2 = -dU dV + dX^i dX^i, \quad (2.29)$$

by the transformation

$$U = \frac{\epsilon}{2\bar{c}} \{e^{2\epsilon\bar{c}\bar{u}} - 1\}, \quad (2.30)$$

$$V = \bar{v} - \bar{c}(1-\epsilon)e^{(1-\epsilon)\bar{c}\bar{u}} X^i X^i, \quad (2.31)$$

$$\bar{x}^i = X^i e^{-(1-\epsilon)\bar{c}\bar{u}}, \quad (2.32)$$

where $\epsilon = \text{sign}(\bar{u})$. We conclude from Eq. (2.28) that an Aichelburg-Sexl boost of a planar domain wall in the direction transverse to the wall results in a planar null shell whose history is the null hypersurface $\bar{u} = 0$ in otherwise flat space-time and with energy density $\bar{\sigma} = \bar{c}(n-1)/4\pi$. The null shell is not accompanied by an impulsive gravitational wave since we started with a space-time M which was conformally flat.

We next consider a domain wall in (A)dS space-time. Thus in addition to our assumption that the space-time M be conformally flat we now assume that its Einstein tensor has the form

$$G_{\beta}^{\alpha} = -\Lambda \delta_{\beta}^{\alpha}, \quad (2.33)$$

where Λ is a constant. Using Eq. (2.12) we find that the equation for $V(z)$ in this case reads

$$V'' + V'^2 = -\frac{2}{n(n-1)}\Lambda. \quad (2.34)$$

We obtain reflection symmetric solutions, in the same way as in the vacuum case, by first solving this equation and then replacing z by $|z|$ in the solution. We find that if $\Lambda > 0$ then

$$e^V = \alpha \cos A_+ |z| + \beta \sin A_+ |z|, \quad (2.35)$$

$$A_+ = \left[\frac{2\Lambda}{n(n-1)} \right]^{1/2}, \quad (2.36)$$

and if $\Lambda < 0$ then

$$e^V = \alpha \cosh A_- |z| + \beta \sinh A_- |z|, \quad (2.37)$$

$$A_- = \left[-\frac{2\Lambda}{n(n-1)} \right]^{1/2}, \quad (2.38)$$

with α and β real constants. In place of Eq. (2.34) these quantities satisfy

$$V'' + V'^2 = -\frac{2}{n(n-1)}\Lambda + 2A_{\pm} \frac{\beta}{\alpha} \delta(z), \quad (2.39)$$

and

$$G_b^a = -\Lambda \delta_b^a + 2A_{\pm}(n-1) \frac{\beta}{\alpha} \delta(z) \delta_b^a. \quad (2.40)$$

In order to have a positive surface energy density we must take $\alpha\beta < 0$. In addition we find in this case that

$$G_b^a(\gamma) = -\lambda \delta_b^a, \quad (2.41)$$

with

$$\lambda = \frac{1}{2}(n-1)(n-2)A_{\pm}^2(\beta^2 \pm \alpha^2), \quad (2.42)$$

and so the hypersurfaces $z = \text{const}$ have de Sitter geometry provided $|\beta| > |\alpha|$. As in the vacuum case we next make a change of the coordinate z to Z such that

$$dZ = e^{-V} dz. \quad (2.43)$$

Calling $\mathcal{V}(Z) = V(z(Z))$ we find that

$$e^{-\mathcal{V}(Z)} = \frac{\sqrt{\beta^2 \pm \alpha^2} \cosh(A_{\pm} \sqrt{\beta^2 \pm \alpha^2} |Z|) - \beta \sinh(A_{\pm} \sqrt{\beta^2 \pm \alpha^2} |Z|)}{\alpha \sqrt{\beta^2 \pm \alpha^2}}, \quad (2.44)$$

and the line element of M now reads

$$ds^2 = -e^{2\mathcal{V}(Z)}(-dt^2 + dZ^2) + e^{2\mathcal{V}(Z) + 2A_{\pm}\sqrt{\beta^2 \pm \alpha^2}t} dx^i dx^i, \quad (2.45)$$

where, as in the vacuum case, $x^0 = t$. Making the Lorentz boost as before and taking the limit $\chi \rightarrow +\infty$ we obtain

$$ds^2 = -e^{2\bar{\mathcal{V}}(\bar{u})} d\bar{u} d\bar{v} + e^{2\bar{\mathcal{V}}(\bar{u}) - 2\bar{A}_{\pm}\sqrt{\beta^2 \pm \alpha^2}\bar{u}} d\bar{x}^i d\bar{x}^i, \quad (2.46)$$

with

$$\bar{A}_\pm = A_\pm \cosh \chi, \quad \bar{\Lambda} = \Lambda \cosh^2 \chi, \quad (2.47)$$

and

$$e^{-\bar{\chi}(\bar{u})} = \frac{\sqrt{\beta^2 \pm \alpha^2} \cosh(\bar{A}_\pm \sqrt{\beta^2 \pm \alpha^2} |\bar{u}|) - \beta \sinh(\bar{A}_\pm \sqrt{\beta^2 \pm \alpha^2} |\bar{u}|)}{\alpha \sqrt{\beta^2 \pm \alpha^2}}. \quad (2.48)$$

The Einstein tensor in the infinite boost limit vanishes except for

$$\bar{G}_{\bar{u}\bar{u}} = -2 \bar{A}_\pm (n-1) \frac{\alpha \beta}{|\alpha|} \delta(\bar{u}). \quad (2.49)$$

Here again we have arrived at a null shell in flat space-time.

III. BOOSTING COSMIC STRINGS

A cosmic string is a line source characterized by a conical singularity. For an infinite straight cosmic string at rest and extending along the z axis, the metric takes the form

$$ds^2 = -dt^2 + d\rho^2 + a^2 \rho^2 d\phi^2 + dz^2. \quad (3.1)$$

Here, ϕ runs from 0 to 2π , and the parameter a takes account of the conical deficit $\Delta\phi$, given by

$$\frac{\Delta\phi}{2\pi} = 1 - a = 4\mu, \quad (3.2)$$

where μ is the mass per unit length of the string. In terms of the rectangular coordinates

$$x = \rho \cos \phi, \quad y = \rho \sin \phi, \quad (3.3)$$

the metric (3.1) reads

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 - (1-a^2) \frac{(xdy - ydx)^2}{x^2 + y^2}. \quad (3.4)$$

We now boost the string sideways along the x direction, by making the coordinate transformation

$$\begin{aligned} t &= \bar{t} \cosh \chi - \bar{x} \sinh \chi = \frac{\bar{u}}{2} e^\chi + \frac{\bar{v}}{2} e^{-\chi}, \\ x &= \bar{x} \cosh \chi - \bar{t} \sinh \chi = -\frac{\bar{u}}{2} e^\chi + \frac{\bar{v}}{2} e^{-\chi}, \\ y &= \bar{y}, \quad z = \bar{z}. \end{aligned} \quad (3.5)$$

Here $\bar{u} = \bar{t} - \bar{x}$, $\bar{v} = \bar{t} + \bar{x}$ are the plane lightlike coordinates. The flat part of the line element (3.4) (the first four terms) then becomes

$$-d\bar{u}d\bar{v} + d\bar{y}^2 + d\bar{z}^2. \quad (3.6)$$

In the limit ($\chi \rightarrow +\infty$), the boost (3.5) becomes lightlike. On noting that

$$xdy - ydx = \frac{e^\chi}{2} (\bar{y}d\bar{u} - \bar{u}d\bar{y}) + O(e^{-\chi}),$$

$$x^2 + y^2 = \frac{e^{2\chi}}{4} (\bar{u}^2 + b^2) + O(e^{-2\chi}), \quad (3.7)$$

where

$$b^2 = 4 e^{-2\chi} \left(\bar{y}^2 - \frac{1}{2} \bar{u}\bar{v} \right), \quad (3.8)$$

the last term of Eq. (3.4) reduces to

$$-(1-a^2) \frac{(\bar{y}d\bar{u} - \bar{u}d\bar{y})^2}{\bar{u}^2 + b^2} + O(e^{-2\chi}). \quad (3.9)$$

The linear mass density of the string measured in the laboratory frame is

$$\bar{\mu} = \mu \cosh \chi = \frac{1}{4} (1-a) \cosh \chi. \quad (3.10)$$

To obtain a regular limiting geometry we must require $\bar{\mu}$ to remain bounded as $\chi \rightarrow +\infty$. This introduces an additional factor $e^{-\chi}$ into Eq. (3.9) through the coefficient $(1-a^2)$. From the identity

$$\lim_{b \rightarrow 0} \frac{b}{\bar{u}^2 + b^2} = \pi \delta(\bar{u}), \quad (3.11)$$

it follows by Eq. (3.8) that

$$\lim_{\chi \rightarrow +\infty} \frac{e^{-\chi}}{\bar{u}^2 + b^2} = \frac{\pi}{2|\bar{y}|} \delta(\bar{u}). \quad (3.12)$$

Inserting this into Eq. (3.9) and combining with Eq. (3.6), we obtain the final limiting form of the boosted metric

$$d\bar{s}^2 = \lim_{\chi \rightarrow +\infty} ds^2 = -d\bar{u}d\bar{v} + d\bar{y}^2 + d\bar{z}^2 - 8\pi \bar{\mu} |\bar{y}| \delta(\bar{u}) d\bar{u}^2. \quad (3.13)$$

This represents the geometry of an infinite straight lightlike string whose world sheet is the 2-flat

$$\bar{u} = \bar{t} - \bar{x} = 0, \quad \bar{y} = 0, \quad (3.14)$$

and whose mass per unit length measured in the laboratory frame is $\bar{\mu}$.

We shall now show that Eq. (3.13) describes a conical singularity along the \bar{z} axis moving at the speed of light in the \bar{x} direction, with angular deficit $\bar{\Delta}\phi$ related to the line density $\bar{\mu}$ in the laboratory frame by an equation identical in form to Eq. (3.2). There are two ways of approach to this. We can start from the conical curvature singularity of metric (3.1) representing the original timelike string in its rest frame and apply the boost (3.5). Alternatively we can work directly from Eq. (3.13). This is a metric of Kerr-Schild form. Both approaches are of interest and we shall consider them in turn.

In the first approach we begin with the static string line element (3.1) or (3.4). The angular deficit $\Delta\phi$ of a 2-plane S_2 of constant z and t is associated with the distributional curvature

$$^{(2)}R = 2(\Delta\phi)\delta_2, \quad (3.15)$$

where

$$\delta_2 = {}^{(2)}g^{-1/2}\delta(x)\delta(y) = \frac{1}{a}\delta(x)\delta(y), \quad (3.16)$$

is the invariant two-dimensional delta function concentrated at the origin. To check Eq. (3.15), note that for Gaussian polar coordinates

$$ds_{(2)}^2 = d\rho^2 + f^2(\rho, \phi)d\phi^2, \quad (3.17)$$

and the Gaussian curvature is

$$^{(2)}R = -\frac{2}{f}\partial_\rho^2 f(\rho, \phi). \quad (3.18)$$

We temporarily smooth out the conical singularity at $\rho=0$ by choosing f to be any smooth function $f_\epsilon(\rho)$ satisfying the conditions

$$f_\epsilon(\rho) = a\rho \quad (\rho \geq \epsilon), \quad \lim_{\rho \rightarrow 0} \frac{f_\epsilon(\rho)}{\rho} = 1. \quad (3.19)$$

Then

$$\begin{aligned} \int \int {}^{(2)}R {}^{(2)}g^{1/2} d\rho d\phi &= -4\pi \int_0^\epsilon (\partial_\rho^2 f_\epsilon) d\rho = 4\pi(1 - f'_\epsilon(\epsilon)) \\ &= 4\pi(1 - a). \end{aligned} \quad (3.20)$$

The conical deficit $\Delta\phi$ is defined by

$$2\pi - \Delta\phi = \lim_{\epsilon \rightarrow 0} \left(\frac{\text{circumference}}{\text{radius}} \right)_{\rho=\epsilon} = \lim_{\epsilon \rightarrow 0} \frac{f'_\epsilon(\epsilon)}{\epsilon} = 2\pi a. \quad (3.21)$$

Combining these results leads to Eq. (3.15).

The Ricci tensor of the xy plane S_2 is, by Eq. (3.15),

$$^{(2)}R_{ab} = (\Delta\phi) {}^{(2)}g_{ab} \delta_2, \quad (3.22)$$

in which the 2-metric ${}^{(2)}g_{ab}$ can be decomposed as

$$^{(2)}g_{ab} = \rho_{,a}\rho_{,b} + a^2 \rho^2 \phi_{,a}\phi_{,b}. \quad (3.23)$$

Since the extrinsic curvature of S_2 is zero, the four-dimensional Ricci tensor of the metric (3.1) can be read off at once from Eq. (3.22). It is

$$^{(4)}R_{\alpha\beta} = \frac{\Delta\phi}{a} \delta(x)\delta(y) (\rho_{,\alpha}\rho_{,\beta} + a^2 \rho^2 \phi_{,\alpha}\phi_{,\beta}), \quad (3.24)$$

in which Greek indices are 4-dimensional and a comma denotes partial differentiation.

Under the boost (3.5), we have for large χ ,

$$\rho \simeq |x| \simeq \frac{e^\chi}{2} |\bar{u}|, \quad \phi = \tan^{-1} \frac{\bar{y}}{\bar{x}} = O(e^{-\chi}),$$

$$\delta(x) \simeq 2e^{-\chi} \delta(\bar{u}). \quad (3.25)$$

We rescale $\Delta\phi$ to a new parameter $\bar{\Delta}\phi$ by analogy with Eq. (3.10),

$$\bar{\Delta}\phi = (\Delta\phi) \cosh \chi, \quad (3.26)$$

and require $\bar{\Delta}\phi$ to stay bounded as $\chi \rightarrow +\infty$. In this limit the second term of Eq. (3.24) becomes negligible, $a \rightarrow 1$ by Eq. (3.2), and we obtain

$$^{(4)}R_{\alpha\beta} = (\bar{\Delta}\phi) \delta(\bar{u}) \delta(\bar{y}) \bar{u}_{,\alpha} \bar{u}_{,\beta}. \quad (3.27)$$

Thus the stress-energy tensor

$$T_{\alpha\beta} = \frac{\bar{\Delta}\phi}{8\pi} \delta(\bar{u}) \delta(\bar{y}) \bar{u}_{,\alpha} \bar{u}_{,\beta}, \quad (3.28)$$

is indeed that of a distributional lightlike source having as its history in space-time the null 2-flat $\bar{u} = \bar{y} = 0$, and with mass per unit length

$$\bar{\mu} = \frac{\bar{\Delta}\phi}{8\pi}, \quad (3.29)$$

measured in the laboratory frame.

To establish the role of $\bar{\Delta}\phi$ in Eq. (3.27) as an angular deficit in the laboratory frame, consider the 2-space \bar{S}_2 of constant \bar{z} and \bar{t} in the geometry (3.13). Recalling that $\bar{u} = \bar{t} - \bar{x}$, we obtain, from Eq. (3.27),

$$^{(4)}R_{xx} = (\bar{\Delta}\phi) \delta(\bar{x}) \delta(\bar{y}). \quad (3.30)$$

Now, $\bar{u}_{,\alpha}$ is lightlike and geodesic for the flat background of the Kerr-Schild metric (3.13), and therefore retains these

properties with respect to the full metric. From this geodesic property and the Gauss-Codazzi equations it follows that ${}^{(4)}R_{xx} = {}^{(2)}R_{xx}$. Thus

$$\frac{1}{2}{}^{(2)}R = {}^{(2)}R_{xx} = {}^{(4)}R_{xx} = (\bar{\Delta}\phi)\delta(\bar{x})\delta(\bar{y}). \quad (3.31)$$

According to Eq. (3.15), this is indeed the distributional curvature in the $\bar{x}\bar{y}$ -plane corresponding to an angular deficit $\bar{\Delta}\phi$.

The alternative route to these results is to proceed directly from the Kerr-Schild metric (3.13) for the lightlike string. Starting with the line element

$$d\bar{s}^2 = -d\bar{u}d\bar{v} + d\bar{y}^2 + d\bar{z}^2 + 2H(\bar{u}, \bar{v}, \bar{y}, \bar{z})d\bar{u}^2, \quad (3.32)$$

the Ricci tensor is

$${}^{(4)}R_{\alpha\beta} = (8HH_{,\bar{v}\bar{v}} - \nabla^2 H)\bar{u}_{,\alpha}\bar{v}_{\beta} - 4H_{,\bar{v}\bar{v}}\bar{u}_{,(\alpha}\bar{v}_{,\beta)} - 4H_{,\bar{v}a}e_{(\alpha}^{(a)}\bar{u}_{,\beta)}, \quad (3.33)$$

where $\nabla^2 = \partial_{\bar{y}}^2 + \partial_{\bar{z}}^2$, $\bar{x}^a = (\bar{y}, \bar{z})$ and $e_{\alpha}^{(a)} = \partial\bar{x}^a/\partial\bar{x}^{\alpha}$. Inserting $H = -4\pi\bar{\mu}\delta(\bar{y})\delta(\bar{u})$ from Eq. (3.13), so that $\nabla^2 H = -8\pi\bar{\mu}\delta(\bar{y})\delta(\bar{u})$, we obtain

$${}^{(4)}R_{\alpha\beta} = 8\pi\bar{\mu}\delta(\bar{y})\delta(\bar{u})\bar{u}_{,\alpha}\bar{u}_{,\beta}, \quad (3.34)$$

in agreement with Eqs. (3.27) and (3.29).

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